Journal Diffusion Factors and their Mathematical Relations with the Number of Citations and with the Impact Factor

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ABSTRACT

If we fix a citing period and a cited period, the Rowlands Journal Diffusion factor (RJDF) is the number of different citing journals divided by the total number of citations. The Frandsen Journal Diffusion factor (FJDF) is the number of different citing journals divided by the total number of citeable articles. Hence the quotient: diffusion factor of Frandsen divided by the one of Rowlands is the impact factor IF (for the given time periods).
This paper investigates the mathematical properties of RJDF and FJDF in function of the number of citations or of IF and shows the validity of some unexplained claims (based on data) given in "T.F.Frandsen. Journal diffusion factors – a measure of diffusion ? Aslib Proceedings 56(1), 5-11, 2004". We show that, under reasonable mathematical conditions (expressed intuitively in Frandsen (2004)), the RJDF is a convexly decreasing function of the number of citations and a concavely increasing function of IF. We also show that $r(\text{RJDF}, \text{IF}) = 0$ implies $r(\text{FJDF}, \text{IF}) > 0$ where r denotes the correlation coefficient of Pearson.

I. Introduction

Journal diffusion factors have been introduced in Rowlands (2002) in order to provide a new journal indicator that describes how scattered the citations (over the different citing journals) to a given journal are. As always also here we need a normalization factor correcting for the "size" of the given journal. In case of the diffusion factor of Rowlands one uses the total number of citations to this journal. Denoting symbolically the number of different citing journals by T and by C the total number of citations to the cited journal we hence have

$$\text{RJDF} = \frac{T}{C} \qquad (1)$$

Of course, to be able to calculate (1) explicitely one must also indicate two time windows: the citing period that one considers (otherwise stated the considered publication period of the citing journals) and the cited period (otherwise stated the considered publication period of the cited journal – which in fact could also be a set of journals, e.g. a scientific field). If the citing period consists of different years one usually speaks of a diachronous study while if the cited period consists of different years one usually speaks of a synchronous study – see Stinson (1981), Stinson and Lancaster (1987) or Egghe and Rousseau (1990), Frandsen, Rousseau and Rowlands (2005). If both periods consist of different years one could even talk about a diasynchronous study. These time issues are of no importance in this paper and hence we can work with (1) in its full generality for the definition of RJDF.

If we denote by P the total number of citeable articles in the cited journal under focus (in the given time period) then we can define the journal diffusion factor as presented by Frandsen (Frandsen (2004)) and denoted by FJDF:

$$\text{FJDF} = \frac{T}{P} \qquad (2)$$

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It is now clear that

$$\frac{\text{FJDF}}{\text{RJDF}} = \frac{\text{C}}{\text{P}} = \text{IF} \tag{3}$$

the well-known (general) impact factor, being number of citations divided by number of citeable publications (again we use general citing and cited time windows).

In Frandsen (2004) one studies both diffusion factors RJDF and FJDF in an experimental way. At the same time, the author formulates some claims, based on these experiments. They can be summarized as follows:

(i) RJDF is a decreasing function of C; more generally: RJDF is negatively correlated (in the sense of Pearson, cf. Egghe and Rousseau (1990, 2001)) with C;
(ii) FJDF is an increasing function of IF; more generally: FJDF is positively correlated with IF;
(iii) RJDF is not correlated with IF while FJDF is positively correlated with IF (as already mentioned in (ii)).

In this paper we are going to give mathematical explanations of all 3 claims, which is unavailable in Frandsen (2004). Basic in our explanation will be the relation between T and C:

$$\text{T} = \varphi(\text{C}) \tag{4}$$

which will be given in terms of the 3 derivatives φ', φ'' and φ'''. In the next section we will give evidence that the relation between T and C is characterized by $\varphi' > 0$, $\varphi'' < 0$, $\varphi''' > 0$. Based on this we can then show that

$$\text{RJDF} = \psi(\text{C}) = \frac{\varphi(\text{C})}{\text{C}} \tag{5}$$

(by (1) and (4)) is even a convexly decreasing function of C, proving (i) at least for functional relations.

In the third section we give a relatively easy proof (again based on properties of φ) of (ii) (again for functional relations). We can even show that FJDF is a concavely increasing function of IF, of the same shape as φ, defined in (4).

The fourth section then shows the following result:

$$r(RJDF, IF) = 0 \Rightarrow r(FJDF, IF) > 0 \tag{6}$$

which is then a (partial) explanation of (iii). We leave open the other assertions expressed in (i), (ii) and (iii) and describe their possible treatment in a concluding fifth section where we indicate possible definitions of "convex or concave clouds of points".

II. Properties of the function φ and derived properties of the function ψ

II.1 Properties of the function φ

The function φ, expressing the relation between T (the number of different citing journals) and C (the total number of citations) is basic in this paper. It is clear that φ is strictly increasing (hence $\varphi' > 0$): the more citations we have the more different citing journals we have (it cannot be less!). Although, in a discrete model, the limit for T is the total number of different journals (and hence φ = constant from a certain point C on), we can assume (in a continuous model) that φ is strictly increasing if C is strictly increasing. It is also clear that (as expressed intuitively in Frandsen (2004)) the number of different citing journals cannot increase "at a linear pace" (see Frandsen (2004), p. 6). Formulated exactly this means that φ is concavely increasing (hence $\varphi'' < 0$). This suffices to prove (i) (the functional form) but we want to go one step further. It is clear that the graph of $T = \varphi(C)$ has the property that the tangent lines have slopes that decrease faster for small C than for large C, i.e. φ' is convexly decreasing, i.e. $(\varphi')'' = \varphi''' > 0$ (we assume that φ''' exists). Graphically this means that we accept a graph of φ as in Fig. 1 (e.g. $\varphi(x) = \ln(x+1)$ or $\varphi(x) = x^{\alpha}$, $\alpha \in]0,1[$ – note that $T = \varphi(c)$ has also the property that $\varphi(0) = 0$ obviously) and not a graph as in Fig. 2 (e.g. $\varphi(x) = \sin x$, for $x \in [0, a]$ with $a \leq \frac{\pi}{2}$).

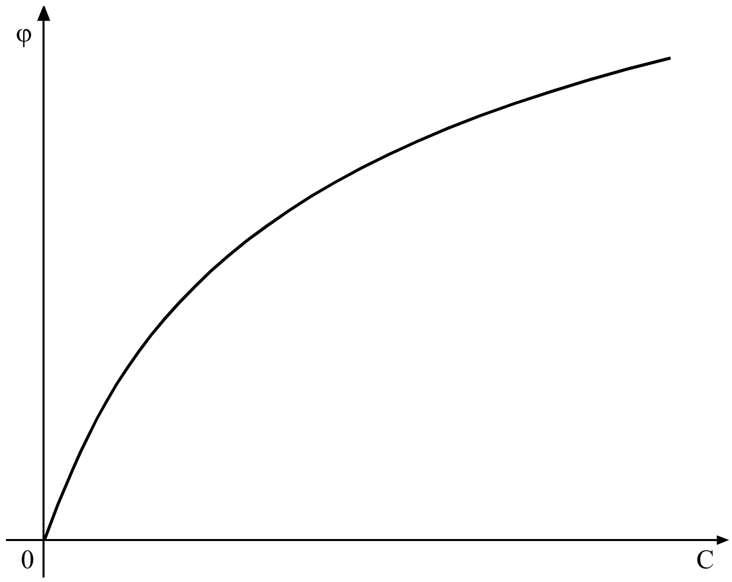

Fig. 1 Acceptable graph of φ : concavely increasing with $\varphi''' > 0$.

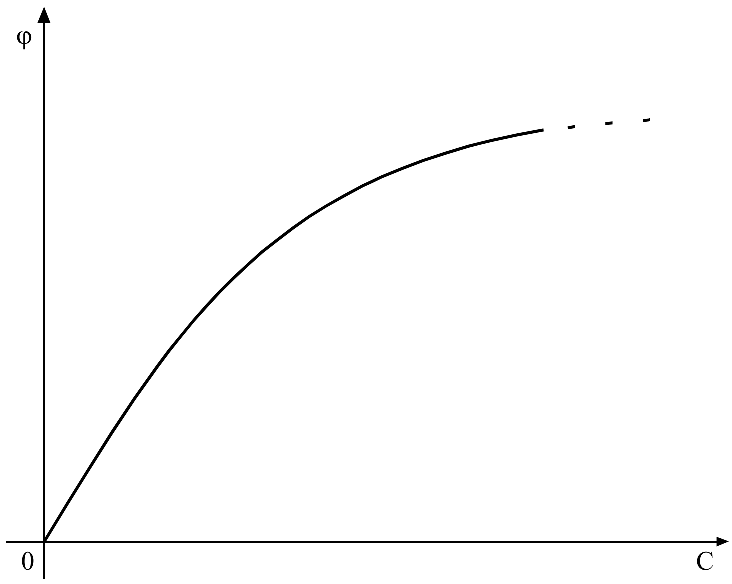

Fig. 2. Not-acceptable graph of φ : concavely increasing with (at least to start with) $\varphi''' < 0$.

We leave it to the reader to check that for $\varphi(x) = \ln(x+1)$ and for $\varphi(x) = x^{\alpha}$, $\alpha \in]0,1[$ that $\varphi' > 0$, $\varphi'' < 0$, $\varphi''' > 0$ while for $\varphi(x) = \sin x$ (for $x \in \left]0, \frac{\pi}{2}\right[$) we have $\varphi' > 0$, $\varphi'' < 0$, $\varphi''' < 0$. To the best of my knowledge this is the first direct application of properties of a third derivative in informetrics: requiring $\varphi' > 0$, $\varphi'' < 0$, $\varphi''' > 0$.

We now have enough machinery to prove some properties of the function ψ in function of C, hence of RJDF in function of C.

II.2 Properties of the function ψ, hence of the relationship between RJDF (Rowland's journal diffusion factor) and C (the total number of citations)

The relationship (5) can now be studied. We have

$$\psi'(C) = \frac{C\varphi'(C) - \varphi(C)}{C^2} < 0 \tag{7}$$

if and only if

$$\varphi'(C) < \frac{\varphi(C)}{C} \tag{8}$$

This inequality can be proved based on the property of φ: concavely increasing (i.e. $\varphi' > 0$, $\varphi'' < 0$) and the fact that $\varphi(0) = 0$: see Fig. 3.

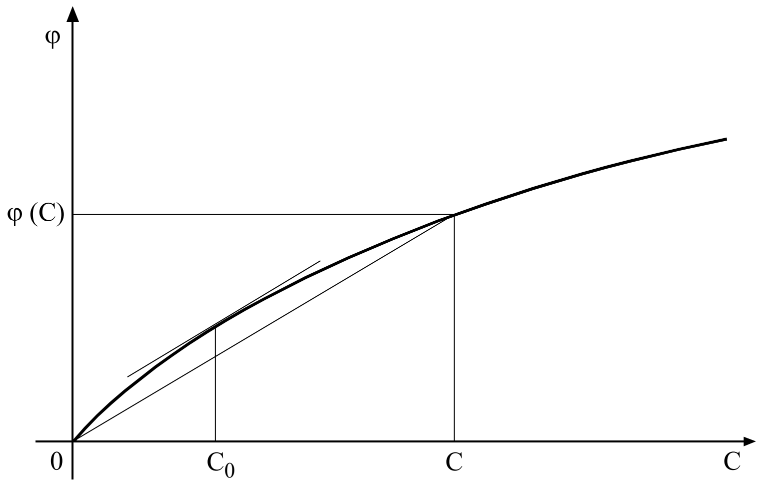

Fig. 3 Proof of (7) using the mean value theorem and that φ is concavely increasing.

We indeed have that the line segment connecting 0 and $(C, \varphi(C))$ (for every $C > 0$) is a chord of the graph $(C, \varphi(C))$. Hence, by the mean value theorem, this chord has a slope equal to the slope of the tangent line to φ in a point $C_0 \in]0, C[$. In other words

$$\varphi'(C_0) = \frac{\varphi(C)}{C} \tag{9}$$

But φ is concavely increasing. Hence

$$\varphi'(C) < \varphi'(C_0) \tag{10}$$

Hence, (9) and (10) yield (8). This proves that ψ, hence RJDF, is a strictly decreasing function of C. This proves (i) for functions.

We can even go further by examining ψ'': based on (7) we have

$$\psi''(C) = \frac{C^2\varphi''(C) - 2C\varphi'(C) + 2\varphi(C)}{C^3} \tag{11}$$

For a convexly decreasing function ψ we hence need to prove that $\psi'' > 0$ hence

$$C^2\varphi''(C) + 2\varphi(C) \geq 2C\varphi'(C) \tag{12}$$

We will now prove (12) using that $\varphi''' > 0$ and $\varphi(0) = 0$. The Taylor expansion of φ (up to φ''' in the rest term) is (for any $x \geq 0$ and $C > 0$):

$$\varphi(x) = \varphi(C) + \varphi'(C)(x - C) + \frac{\varphi''(C)}{2!}(x - C)^2 + \frac{\varphi'''(C_0)}{3!}(x - C)^3 \tag{13}$$

for $\exists C_0 \in]x, C[$, $(x < C)$ or $]C, x[$, $(x > C)$.

Taking $x = 0$ we have, by (13)

$$0 = \varphi(0) = \varphi(C) - C\varphi'(C) + \frac{\varphi''(C)}{2}C^2 - \frac{\varphi'''(C_0)}{3!}C^3 \tag{14}$$

So

$$\varphi(C) - C\varphi'(C) + \frac{\varphi''(C)}{2}C^2 = \frac{\varphi'''(C_0)}{3!}C^3 > 0$$

, since $\varphi''' > 0$. This proves (12). It might seem remarkable that properties of ψ' are proved using properties of φ', φ'' and that in ψ'' a property of φ''' is involved. However this is not surprising since $\psi(x) = \frac{\varphi(x)}{x}$, being the slope of all chords through 0 in the graph of φ which is more related with φ' than with φ. Nevertheless finding (12) via a Taylor expansion and $\varphi''' > 0$ is surprising. Note also that (12) is a stronger version of (8) since $\varphi'' < 0$.

Theorem II.1:
RJDF is a convexly decreasing function of C. (See Fig. 4).

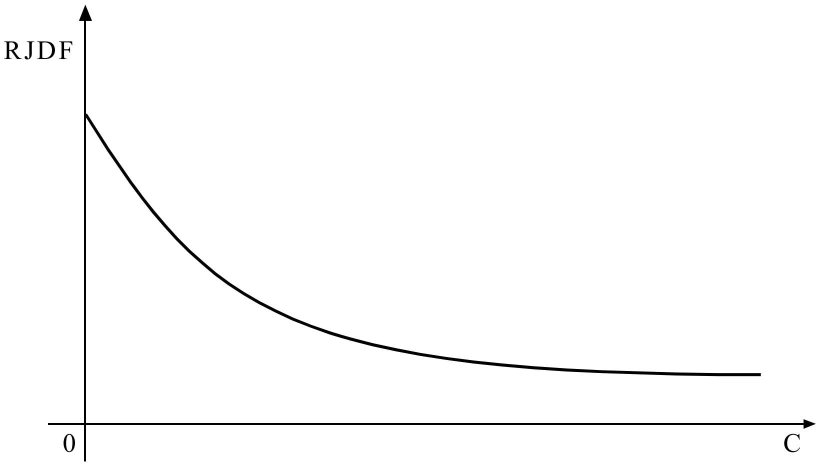

Fig. 4 Relationship between RJDF in function of C: functional solution of claim (i).

Note that (as is readily seen, using de l'Hôspital's rule, (7) and the fact that $\varphi(0) = 0$)

$$\psi'(0) = \frac{\varphi''(0)}{2} < 0 \tag{15}$$

The behavior of $\psi(C)$ for large values of C is more or less clear: or we have that

$$\lim_{C\to\infty} \varphi(C) < \infty \tag{16}$$

yielding

$$\lim_{C\to\infty} \psi(C) = \lim_{C\to\infty} \frac{\varphi(C)}{C} = 0,$$

or we have that (16) is not true (hence we have that $\lim_{C\to\infty} \varphi(C) = \infty$) in which case we have, using the l'Hôspital's rule, that

$$\lim_{C\to\infty} \psi(C) = \lim_{C\to\infty} \frac{\varphi(C)}{C} = \lim_{C\to\infty} \varphi'(C). \tag{17}$$

We can easily assume that

$$\lim_{C\to\infty} \varphi'(C) = 0 \tag{18}$$

indicating the fast decrease of $T = \varphi(C) =$ # different citing journals with respect to C = # citations. Hence

$$\lim_{C\to\infty} \psi(C) = 0 \tag{19}$$

naturally.

Note:
It is clear from the definition of φ that $\varphi(C) \le C$ for all $C > 0$ (since $T \le C$ obviously) (hence the graph of Fig. 3) but we did not use this in the derivation above. Obviously $\varphi(C) \le C$ implies $\psi(C) = RJDF \le 1$, always !

This ends the treatment of the relationship between RJDF and C as expressed in (i) (functional form). In the next section we turn our attention to claim (ii), again using the function φ.

III. The relationship between FJDF (Frandsen's journal diffusion factor) and IF (the impact factor)

The only extra assumption we use in this section is that the impact factor IF is independent of P, the number of articles in the journal. Indeed IF is a measure of relative citation degree of an article: it normalises C (the total number of citations to a journal) by dividing by P (i.e. the size of the journal). At least theoretically this shows that IF is independent of P (in practise, different types of journals (e.g. review journals) might show some heterogenity with respect to this property but we can use the assumption that IF is independent of P as a first approximation).

We have the following

Theorem III.1:
FJDF is a concavely increasing function of IF.

Proof:
By (3) and (5) we have

$$\mathrm{FJDF} = \mathrm{RJDF.IF}$$

$$\mathrm{FJDF} = \frac{\varphi(\mathrm{C})}{\mathrm{C}}\frac{\mathrm{C}}{\mathrm{P}}$$

$$\mathrm{FJDF} = \frac{\varphi(\mathrm{IF.P})}{\mathrm{IF.P}}\mathrm{IF}$$

$$\mathrm{FJDF} = \frac{\varphi(\mathrm{IF.P})}{\mathrm{P}} \tag{20}$$

Hence, using the above assumption, (20) gives

$$\frac{\mathrm{d}(\mathrm{FJDF})}{\mathrm{d}(\mathrm{IF})} = \frac{1}{\mathrm{P}}\frac{\mathrm{d}\varphi(\mathrm{IF.P})}{\mathrm{d}(\mathrm{IF.P})}\frac{\mathrm{d}(\mathrm{IF.P})}{\mathrm{d}(\mathrm{IF})}$$

$$= \varphi'(\mathrm{x}) > 0 \tag{21}$$

in the point $\mathrm{x} = \mathrm{IF.P}$. Further, using (21)

$$\frac{d^2(FJDF)}{d(IF)^2} = \frac{d}{d(IF)}\left[\frac{d\varphi(IF.P)}{d(IF.P)}\right]$$

$$= \frac{d}{d(IF.P)}\left[\frac{d\varphi(IF.P)}{d(IF.P)}\right]\frac{d(IF.P)}{d(IF)}$$

$$= \mathrm{P}\varphi''(\mathrm{x}) < 0 \tag{22}$$

in the point $\mathrm{x} = \mathrm{IF.P}$. □

Finally we turn our attention to claim (iii) in the Introduction.

IV. Relations between r(RJDF,IF) and r(FJDF,IF) (r = Pearson correlation coeeficient)

This section works with the full generality of correlations in the sense of Pearson as is the case in the claims of Frandsen (2004). We have the following

<u>Theorem IV.1</u>:

$$r(RJDF, IF) \geq 0 \Rightarrow r(FJDF, IF) > 0 \quad (23)$$

Proof:
This theorem only uses that

$$\text{FJDF} = \text{RJDF.IF} \quad (24)$$

and not the relations (1) or (2). Using the formula for the correlation coefficient of Pearson (see e.g. Egghe and Rousseau (1990, 2001) or any book on statistics) we have

$$r(RJDF, IF) = \frac{\frac{1}{N}\sum_{j=1}^{N} x_j y_j - \frac{1}{N}\sum_{j=1}^{N} x_j \frac{1}{N}\sum_{j=1}^{N} y_j}{\sqrt{\left(\frac{1}{N}\sum_{j=1}^{N} x_j^2 - \left(\frac{1}{N}\sum_{j=1}^{N} x_j\right)^2\right)\left(\frac{1}{N}\sum_{j=1}^{N} y_j^2 - \left(\frac{1}{N}\sum_{j=1}^{N} y_j\right)^2\right)}} \quad (25)$$

, denoting $(x_j, y_j) = (IF_j, RJDF_j)$, where $y_j = RJDF_j =$ the Rowlands journal diffusion factor of journal j and $x_j = IF_j =$ the impact factor of journal j (same cited and citing periods as described in the Introduction), $j = 1,...,N$ (an arbitrary set of N journals).

Using the same notation and (24) we also have

$$r(FJDF, IF) = \frac{\frac{1}{N}\sum_{j=1}^{N} x_j^2 y_j - \frac{1}{N}\sum_{j=1}^{N} x_j \frac{1}{N}\sum_{j=1}^{N} x_j y_j}{\sqrt{\left(\frac{1}{N}\sum_{j=1}^{N} x_j^2 - \left(\frac{1}{N}\sum_{j=1}^{N} x_j\right)^2\right)\left(\frac{1}{N}\sum_{j=1}^{N} x_j^2 y_j^2 - \left(\frac{1}{N}\sum_{j=1}^{N} x_j y_j\right)^2\right)}} \quad (26)$$

Given that $r(RJDF, IF) \geq 0$ we have, by (25)

$$\sum_{j=1}^{N} x_j y_j \geq \frac{1}{N}\sum_{j=1}^{N} x_j \sum_{j=1}^{N} y_j \quad (27)$$

In order to prove (23) we must show that, using (26),

$$\sum_{j=1}^{N} x_j^2 y_j > \frac{1}{N}\sum_{j=1}^{N} x_j \sum_{j=1}^{N} x_j y_j \quad (28)$$

The inequality of Cauchy-Schwartz (see e.g. Apostol (1957) or Protter and Morrey (1977)) yields

$$\left(\sum_{j=1}^{N} x_j y_j\right)^2 = \left(\sum_{j=1}^{N} x_j \sqrt{y_j}\sqrt{y_j}\right)^2$$

$$< \sum_{j=1}^{N} x_j^2 y_j \sum_{j=1}^{N} y_j \qquad (29)$$

except if there exists a constant $a \in \Re$ such that

$$x_j\sqrt{y_j} = a\sqrt{y_j} \qquad (30)$$

for all $j = 1,...,N$ (then there is equality in (29)). But (30) requires that all $x_j = IF_j$ are the same (which we exclude – see also the Note below). By (29) we have

$$\sum_{j=1}^{N} x_j^2 y_j > \frac{\left(\sum_{j=1}^{N} x_j y_j\right)^2}{\sum_{j=1}^{N} y_j}$$

$$= \sum_{j=1}^{N} x_j y_j \frac{\sum_{j=1}^{N} x_j y_j}{\sum_{j=1}^{N} y_j}$$

$$\geq \frac{1}{N}\sum_{j=1}^{N} x_j y_j \sum_{j=1}^{N} x_j$$

, by (27), hence proving the strict inequality (28). □

Note:

If all impact factors are equal (which we excluded in the above proof) both clouds of points $\left(IF_j, RJDF_j\right)_{j=1}^{N}$ and $\left(IF_j, FJDF_j\right)_{j=1}^{N}$ are situated on a single vertical straight line with abscis $x = a$ (notation of (30)). Hence their correlation coefficients are undefined $\left(\frac{0}{0}\right)$ and can be defined as 1 which then makes that Theorem IV.1 is also valid in this very special (deteriorated) case.

As a trivial corollary we find a partial explanation for claim 3 in the Introduction.

Corollary IV.2:

$$r(RJDF, IF) = 0 \Rightarrow r(FJDF, IF) > 0 \qquad (31)$$

V. Conclusions and open problems

In this paper we defined the function $T = \varphi(C)$ being the number of different citing journals in function of the total number of citations. It is found that φ has the characteristic properties: $\varphi' > 0$, $\varphi'' < 0$, $\varphi''' > 0$. Based on this we show that $\psi(C) = \frac{\varphi(C)}{C}$, being the Rowlands journal diffusion factor RJDF, is a convexly decreasing function of C. We leave open whether (in general) RJDF is negatively correlated with C (in the sense of Pearson) as claimed in Frandsen (2004). The above is a partial proof of this claim.

Within this framework we also show that the Frandsen journal diffusion factor, FJDF, is a concavely increasing function of IF (the impact factor). We leave open whether FJDF (in general) is positively correlated with IF and whether this follows from the claimed negative correlation of RJDF with C. The above is a partial solution to the second claim.

We also show that

$$r(RJDF, IF) \geq 0 \Rightarrow r(FJDF, IF) > 0$$

so that we have that if RJDF and IF are not correlated (we do not know if this is generally true) then FJDF and IF are positively correlated, a partial proof of claim (iii).

We hope this paper sheds some light on the relations between RJDF and FJDF with respect to C and IF. Mathematically we feel that the general claims of Frandsen (2004), using Pearson's correlation coefficient, are not entirely true (in the present general formulation). In a thorough treatment of these problems we must extend the results obtained in this paper on the functions φ and ψ. Therefore we need to extend the results on concave and convex <u>functions</u> to concave and convex "<u>clouds of points</u>" for which a definition still must be produced. In this concluding section we want to propose a definition in order to show the reader in what direction we are thinking. As the reader will notice, the proposal is not readily useable and hence further development of these views is needed.

Let us have a general cloud of points $\{(x_1, y_1), (x_2, y_2), ..., (x_N, y_N)\}$. To determine whether this cloud increases or decreases (in the sense of regression lines) it is well-known that the calculation of the correlation coefficient of Pearson r yields an answer: increasing if $r > 0$, decreasing if $r < 0$. Now convexity or concavity for curves is a matter of increase or decrease of the first derivative. Therefore we can propose the following definitions.

<u>Definitions V.1</u>:
Let us have a cloud of points as above. For each $i = 1, ..., N - 1$, replace y_i by

$$z_i = \frac{y_{i+1} - y_i}{x_{i+1} - x_i} \tag{32}$$

, i.e. replacing each point (x_i, y_i) by (x_i, z_i) indicating in each abscissa x_i the value of the slope of the connecting line between (x_i, y_i) and (x_{i+1}, y_{i+1}), $i = 1, ..., N - 1$.

Let now r^* be the correlation coefficient of Pearson for the new cloud of points $\{(x_1, z_1), ..., (x_{N-1}, z_{N-1})\}$. We say that the original cloud of points is convex if $r^* > 0$. If $r^* < 0$ we say that the original cloud of points is concave. By the definition of the correlation coefficient of Pearson we hence have convexity (respectively concavity) iff

$$\sum_{j=1}^{N-1} x_j \frac{y_{j+1} - y_j}{x_{j+1} - x_j} - \frac{1}{N-1} \sum_{j=1}^{N-1} x_j \sum_{j=1}^{N-1} \frac{y_{j+1} - y_j}{x_{j+1} - x_j} \tag{33}$$

is > 0 (respectively < 0).

We leave it as an open problem to create more workable definitions of convexity or concavity of clouds of points. Then, with these notions, one can then try to extend the function theoretical results (on φ and ψ) to these general clouds, hereby shedding more light to the general claims of Frandsen (2004), but now also including properties on concavity or convexity of the respective clouds of points.

Let us give one example: the first claim of Frandsen (2004). Let T_j, C_j denote the T, C values defined in the Introduction for journal $j = 1,...,N$. To prove that RJDF is negatively correlated with C, based on the positive correlation of T and C (i.e. the extension of the proof of properties of the function ψ based on those of the function φ) we need to prove (use the numerators of the correlation coefficients, the denominators being positive)

$$N\sum_{j=1}^{N} T_j C_j \geq \sum_{j=1}^{N} T_j \sum_{j=1}^{N} C_j$$

$$\Rightarrow N\frac{\sum_{j=1}^{N} T_j}{\sum_{j=1}^{N} C_j} \leq \sum_{j=1}^{N} \frac{T_j}{C_j} \qquad (34)$$

Note that in the case of a decreasing cloud of points $\left\{\left(C_1, \frac{T_1}{C_1}\right), \ldots, \left(C_N, \frac{T_N}{C_N}\right)\right\}$, inequality (34) is valid since decreasing clouds of points have a negative correlation coefficient of Pearson (see e.g. Egghe and Rousseau (1996) for a proof of this well-known fact). This case is, however, described in the proof of Theorem II.1: ψ decreases if φ concavely increases. A direct proof of (34) is not possible since, in the simple case of deriving properties of ψ from those of φ we needed concavity of φ to show that ψ decreases (since $\psi(x) = \frac{\varphi(x)}{x}$). A concrete example (communicated to me by R. Rousseau to whom my sincerest thanks) is given as follows: let us have the data as in Table 1

Table 1. C, T data contradicting implication (34)

j	C_j	T_j	$\frac{T_j}{C_j}$
1	1	10	10
2	2	150	75
3	3	151	50.333
4	4	200	50

Here $r(T,C) = 0.900117 > 0$ while also $r(RJDF,C) = 0.457474 > 0$. We can even make $r(RJDF,C) > r(T,C) > 0$ as the next example shows (also communicated to me by R. Rousseau).

Table 2. C, T data contradicting implication (34) such that $r(RJDF,C) > r(T,C) > 0$

j	C_j	T_j	$\frac{T_j}{C_j}$
1	100	1	0.01
2	300	150	0.5
3	301	151	0.501661
4	302	200	0.662252

Here $r(RJDF,C) = 0.964975 > r(T,C) = 0.96456$.

It is easy to see that in both examples, the relation $\varphi : C_j \to T_j$ is not concave, otherwise a counterexample would not have been possible, due to Theorem II.1.

For the same reason we conjecture that (34) can be proved using "concavity" of the cloud of points $\{(C_1, T_1), ..., (C_N, T_N)\}$.

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