Power-law Fractal: The Law of Technological Innovation Output Statistics[1]

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Abstract
With patent quantity and GDP data, the author researched the quantitative relationship between patent quantity, which is an output index of technological innovation, and Gross Domestic Product, which stands for the economic output of one country or region. Based on the analysis and calculation of 6 years' data (from 1998 to 2003) of certain countries with high patent output, the author found out there is a strong correlation (the fitting correlation coefficient is over 0.9) between patent output and GDP of many developed countries. This is a Fractal, quantitative power function relation. In this article, the author is discussing about the properties and meaning of this fractal, as well as the method of applying this quantitative relation for analyzing the technological innovation level and efficiency of one country or region, and predicting its patent quantity.

Introduction
The science and technological progress is an important propelling force for the world economy development and the society improvement. The progress of science and technology is closely related to the country's or region's economic situation. Generally, in the developed country or region, the more the science and technology input, the more the science and technology output, and the higher the technological innovation efficiency. Also, the stronger the scientific and technological ability, the higher the technological innovation efficiency. Thus the bigger the innovation output, the more well-developed the economy. Therefore, we usually use the output of science and technology dissertations and patents to measure the scientific and technological strength and development quantitatively and qualitatively. Dissertations and the relevant publication can present the results from science and technology research, especially from the basic research part, while patent is the 'visible proof' of technology innovation and invention. At present, there are plenty of statistical analyses about dissertations, particularly influencing the science and technology output and evaluation greatly. Internationally, patent measurement has been studied as an index of technological innovation or improvement and economic indicator for many years already (Griliches, 1990). This is an important indicator recommended by many economic professionals. Actually a lot of scholars (Comanor, Scherer, 1969, Basberg, 1987, Narin, Noma, Perry, 1987, Archibugi, 1992, Grupp, Schmooh, 1999, Rozhkov, Ivantcheva, 1998, Fung, Chow, 2002, Geisler 2000) have been doing research about how to use patent to weigh up science technology level and to connect economic activities, including measuring one company's technical inferiority and superiority of science and technology investment combination and the intensity of knowledge flow with patent statistics and the associated analysis; studying the function of how patents supporting enterprises with economical repayment; researching technical intensity, technical change and technological diffusion with patent statistics; analyzing patents' value; forecasting the quantity of patent application; evaluating organization's technological innovation, and so on.

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Although same as bibliometrics, patent statistical analysis has its own internal drawback, its data are more complete and covering longer term, also available for study and comparison different layers of countries, regions, industries and organizations. Hence it is used in various evaluation and decision activities in research organizations, enterprises, universities and governments, and other institutes. For example, US National Science Foundation (NSF) has precisely applied patent statistical analysis and assessment in its once-two-year 'Science & Engineering Indicators' reports. This article is going to make use of data of patent and GDP to do empirical research on the relationship between the economy output and the technological innovation output from a macroscopic perspective at the national aspect.

Derek De Solla Price (1963), a famous American scientist, is the first to explore the output of science research (quantity of dissertation) in the economic activities. He pointed out that the amount of scientific publication coming from a country was essentially proportional to its economic size, as measured by its gross domestic product, and not proportional to geographic area, population or any other parameters. From Prof. Price's view, the science research output is a characteristic of a modern country, and it's positively related to this country's economic size.

Prof. F. Narin (1994), one famous American patent statistics analyst, had proved the patent bibliometrics is very similar as the literature bibliometrics through his study, which means the statistical law for literature bibliometrics should also be suitable for patent bibliometrics. He pointed out that Price's point holds not only for scientific publication, but also for patenting. When did research about national productivity, he had made a logarithm diagram with 18 countries' US patent quantities and the relevant GDP data in 1987, and found these dots are approximately on one straight line, with correlation. Thus he drew the conclusion that measuring one country's science and technology productivity with its papers and patents has approximately correlation with this country's economic activities.

Although both of Prof. Price and Prof. Narin has found out that science and technology dissertation and patent output is closed related with the country's economic size, which is qualitatively defined as the bigger the economic size, the more the dissertation and patent output, they did not go further to set up a specific and quantitative equation for these factors. Moreover they did not answer following questions: this phenomenon exists generally? Also exists on the level of regions besides that of countries? What is the real reason and essence of it? Therefore, we are going to discover answers to these questions. Below, we will use 6 years' data (PCT & US Patent) to process empirical study.

Method & Data

Method

Take the logarithm of country's patent as the Y-coordinate, and take the logarithm of that country's GDP as the X-coordinate. After getting the scatter dots diagram, observe its characteristics. If there is correlation, use 6 year data of these countries (or take several years' data depending on the specific situation) and apply SPSS statistics software for fitting computation, in order to obtain relevant parameters and quantitative functional relation.

Data

Select world patent (PCT) data which were released by WIPO Patent Bureau, and US Patent data which were approved by United States Patent and Trademark Office. These data are from the websites of WIPO (http://www.wipo.int/pct/en/activity) and United States Patent and Trademark Office (http://www.uspto.gov/). And GDP data are from the World Economic Outlook Database of International Monetary Fund. From high to low, we choose sample data of top 20 countries for our research (see to Table 1 and Table 2). From these 2 tables, it is very easy to notice that the top 10 countries with most PCT output, whose research investment is over 1.8% of their GDP, have more than 90% PCT output over the world.

Table 1*: 1998–2003 Number of World Patent & Data of GDP for Some Countries

Year	**1998**		**1999**		**2000**		**2001**		**2002**		**2003**	
Country	World Patent	GDP Million US$	World Patent	GDP Million US$	World Patent	GDP Million US$	World Patent	GDP Million US$	World Patent	GDP Million US$	World Patent	GDP Million US$
USA	28 356	8 781 525	29 463	9 274 325	38 171	9 824 650	40 003	10 082 150	44 609	10 446 250	39 250	10 881 609
Germany	9 112	2 147 437	10 897	2 110 841	12 039	1 875 162	13 616	1 857 345	15 269	1 992 339	13 979	2 400 655
Japan	6 098	3 945 572	7 255	4 473 386	9 402	4 766 108	11 846	4 175 922	13 531	3 986 347	16 774	4 326 444
UK	4 383	1 423 494	4 741	1 460 381	5 538	1 440 929	6 233	1 430 063	6 274	1 566 748	6 090	1 794 858
France	3 322	1 454 326	3 633	1 444 489	3 601	1 313 303	4 619	1 321 902	4 877	1 437 377	4 723	1 747 973
Netherlands	2 065	394 002	2 153	399 071	2 587	371 606	3 187	384 357	4 019	419 774	4 180	511 556
Sweden	2 554	248 287	2 619	251 566	3 071	239 763	3 502	219 439	2 988	240 312	2 491	300 795
Korea	485	317 079	790	406 071	1 514	461 520	2 318	427 235	2 552	476 690	2 947	605 331
Swiss	1 293	262 576	1 564	258 987	1 701	240 432	2 011	246 109	2 469	268 366	2 379	309 405
Canada	1 315	616 782	1 398	661 251	1 600	724 233	2 030	715 061	2 210	735 956	2 102	834 390
Australia	1 048	363 355	1 154	391 903	1 627	378 741	1 754	357 616	1 775	398 157	1 729	518 382
Italy	925	1 198 183	1 130	1 182 048	1 354	1 077 571	1 574	1 092 851	2 041	1 187 953	2 023	1 465 895
Finland	1 092	129 194	1 269	128 538	1 437	121 220	1 623	120 967	1 762	131 163	1 497	161 549
Israel	672	102 817	729	103 108	924	113 907	1 248	111 791	1 199	102 707	1 161	103 689
Denmark	624	172 428	792	173 123	789	158 451	929	159 234	989	172 928	1 021	212 404
China	322	946 310	240	991 351	579	1 080 764	1 670	1 175 842	1 124	1 266 054	1 205	1 409 852
Spain	378	588 779	457	602 975	519	562 842	575	583 656	729	655 105	776	836 100
Russia Federation	429	270 951	532	195 906	590	259 716	551	309 921	616	346 535	527	433 491
Belgium	428	250 321	513	251 125	574	228 295	681	227 112	697	244 704	725	302 217
Austria	421	212 192	432	210 330	476	191 246	563	189 754	563	204 701	620	251 456
Norway	394	150 049	436	158 099	470	166 905	525	169 780	525	190 477	448	221 579
South Africa		133 583	281	130 980	386	128 049	418	114 255	407	104 475	376	159 886
New Zealand	178	54 245	242	55 960	264	51 421	279	50 491	301	58 157	296	76 526
Singapore	127	81 910	144	81 380	225	91 476	271	84 871	322	86 969	313	91 342
India		409 422	61	436 798	156	460 792	316	476 119	480	494 821	611	598 966
Ireland	150	87 070	167	95 603	184	95 001	212	102 772	257	122 108	237	148 553

***Resources from:** Number of World Patent (PCT): http://www.wipo.int/pct/en/activity/
GDP Data: International Monetary Fund, World Economic Outlook Database:
http://www.atkearney.com/shared_res/pdf/GDP_data_2004_S.pdf,
GDP 2003: http://www.worldbank.org/data/databytopic/GDP.pdf
GDP of Belgium: http://unstats.un.org/unsd/snaama/resultsGDP.asp , and
http://unstats.un.org/unsd/snaama/resultsGDP.asp

Table 2*: 1998-2003 Number of Assigned U.S. Patent & GDP Data for Some Countries

Year	**1998**		**1999**		**2000**		**2001**		**2002**		**2003**	
Country	US Patents	GDP Million US$	US Patents	GDP Million US$	US Patents	GDP Million US$	US Patents	GDP Million US$	US Patents	GDP Million US$	US Patents	GDP Million US$
USA	80 294	8 781 525	83 908	9 274 325	85 072	9 824 650	87 607	10 082 150	86 977	10 446 250	87 901	10 881 609
Japan	30 841	3 945 572	31 104	4 473 386	31 296	4 766 108	33 223	4 175 922	34 859	3 986 347	35 517	4 326 444
Germany	9 095	2 147 437	9 337	2 110 841	10 234	1 875 162	11 260	1 857 354	11 277	1 922 339	11 444	2 400 655
France	3 674	1 454 326	3 820	1 444 489	3 819	1 313 303	4 041	1 321 902	4 035	1 437 377	3 869	1 747 973
UK	3 464	1 423 494	3 572	1 460 381	3 667	1 440 929	3 965	1 430 063	3 838	1 566 748	3 627	1 794 858
Korea	3 259	317 079	3 562	406 071	3 314	461 520	3 538	427 235	3 786	476 690	3 944	605 331
Canada	2 974	616 782	3 226	661 251	3 419	724 233	3 606	715 061	3 431	735 956	3 426	834 390
Italy	1 582	1 198 183	1 492	1 182 048	1 714	1 077 571	1 709	1 092 851	1 750	1 187 953	1 722	1 465 895
Swiss	1 278	262 576	1 280	258 987	1 322	240 432	1 420	246 109	1 364	268 366	1 308	309 465
Netherlands	1 226	394 002	1 247	399 071	1 241	371 606	1 332	384 357	1 391	419 774	1 325	511 556
Sweden	1 225	248 287	1 401	251 566	1 577	239 763	1 743	219 439	1 675	240 312	1 521	300 795
Israel	754	102 817	743	103 108	783	113 907	970	111 791	1 040	102 707	1 193	103 689
Australia	720	363 355	707	391 930	704	378 741	875	357 616	858	398 157	900	518 382
Belgium	693	250 321	648	251 125	694	228 295	718	227 112	722	244 704	622	302 217
Finland	595	129 194	649	128 538	618	121 220	732	120 967	809	131 163	865	161 549
Denmark	391	172 428	487	173 123	436	158 451	479	159 234	426	172 928	529	212 404

***Resources from:**
United States Patent and Trademark Office: http://www.uspto.gov/go/taf/topo_98.pdf - /topo_03.pdf
Data of GDP: http://www.atkearney.com/shared_res/pdf/GDP_data_2004_S.pdf,
http://www.worldbank.org/data/databytopic/GDP.pdf, http://unstats.un.org/unsd/snaama/resultsGDP.asp,
http://unstats.un.org/unsd/snaama/resultsGDP.asp?, http://www.worldbank.org/data/databytopic/GDP.pdf

Calculation Results

The Relationship Between World Patent Quantity & GDP

With data from over 10 developed countries from 1998 to 2003, we present the Fig. 1, which is the scatter dots diagram (left) and fitting computation diagram (right) of the logarithm of world patent (PCT) quantity (Ln(Y=PCT)) and the logarithm of relevant GDP (Ln(X=GDP)).

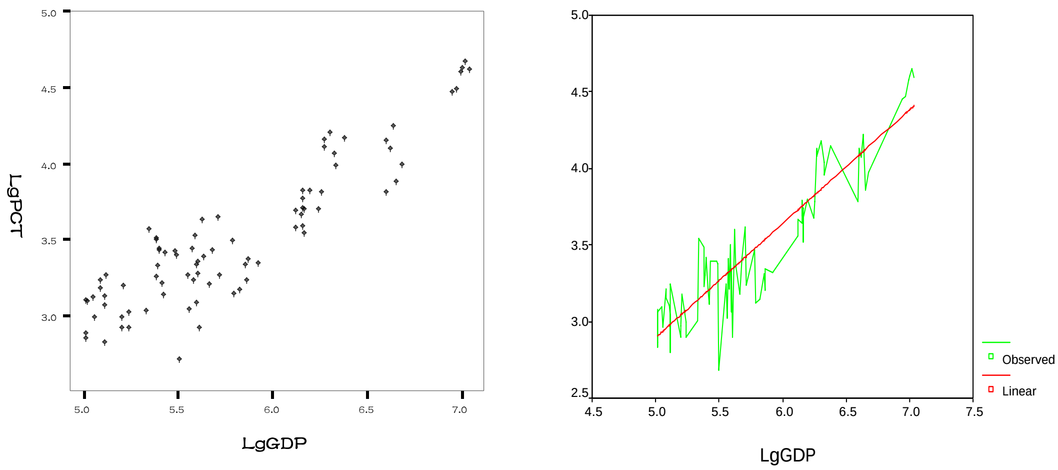

Figure 1: Correlation Analysis of Developed Countries' PCT & GDP

Fig 1 shows that there is a strong correlation between the logarithm of PCT patent quantity and the logarithm of GDP, which is a quantitative functional relationship:

$$LnY(x) = a + bLnx \qquad (1)$$

The actual results of fitting computation are:

Independent: Ln(x=GDP)							
Dependent:	Mth	Rsq	d.f.	F	Sigf	a	b
Ln(Y=PCT):	LIN	0.817	82	366.58	0.000	-0.8253	0.7440

According to these, we know that the correlation coefficient R^2 is 0.817, F criterion is 366.58, Sigf is 0.000, and the size of sample data is 84. These prove that there is a significant correlation between patent quantity and GDP. Here **a** is one constant, which depends on the sample data and **b**, 0.7440, is the straight line's slope, which is in fact a power index. The reason is below. After we equivalently transform the equation (1), we will get:

$$Y(x) = Ax^b \qquad (2)$$

Or

$$Y(x) \% x^b \qquad (3)$$

$$b = LnY(x)/Ln(x) \qquad (4)$$

Equation (2) and (3) show that there is a quantitative power function relationship, not a simple linear function relationship ($b \neq 1$), between patent quantity and GDP. The slope of the double logarithm straight line is b, which is a power index, playing an important role in the relationship. When we use original data to process fitting computation of the power function, we could obtain same b, R^2, F and Sigf as those above.

The point to be explained more clearly here is we have practiced the fitting computation with 14 countries' data, which are US, Germany, Japan, UK, France, Netherlands, Switzerland, Korea, Swiss, Canada, Australia, Finland, Israel and Denmark. We do not include Italy inside, because it is in special situation. Italy's investment to science and technology research and development (R&D) is only about 1% of its GDP, almost as low as those in developing countries, although its GDP is not low at all. Thus Italy's dots are far away from the others' on the scatter dot diagram. On the other hand, these 14 countries have spent a lot on their R&D, at least over 1.8% of their GDP. Therefore, we consider Italy's case as an exception. We also do all of the processes with Italy's data included, and the result is there is still a correlation, although the R^2 has decreased. Since this result matches our prediction, we think it is reasonable to eliminate Italy's data.

The Relationship Between Other Countries' World Patent Quantity & GDP
We also have studied those countries whose patent quantities are not very low but lower than previous countries, such as Spain, Belgium, Austria, Norway, South Africa, Singapore, Russia, China, New Zealand and so no. We found that these countries' dots deviate from previous straight correlation line of developed countries, because they have lower investment to R&D and less world patent quantities. When considering these countries separately from developed countries, we assume that there is no quantitative power function correlation between their Ln(PCT) and Ln(GDP) before 2000 (including 2000), and there is correlation since 2001 for the reason that these countries have increased their world patent quantities. Thus, we analyze these countries' data as below, see to Figure 2.
The outcome is:

Independent: Ln(x=GDP)

Dependent:	Mth	Rsq	d.f.	F	Sigf	a	b
Ln(Y=PCT):	LIN	0.841	25	131.95	0.000	0.3268	0.4475

From this result, we can see that power index b is only 0.4475. Hence our conclusion is, when compared with previous developed countries, these countries belong to another layer or group from the point of technological innovation efficiency or ability. We also can confirm this with the fact that these countries only use 1% or less of GDP on their R&D. Consequently, we think it is not necessary to consider other countries with even weaker science and technology power or economic power and lower PCT patent quantities.

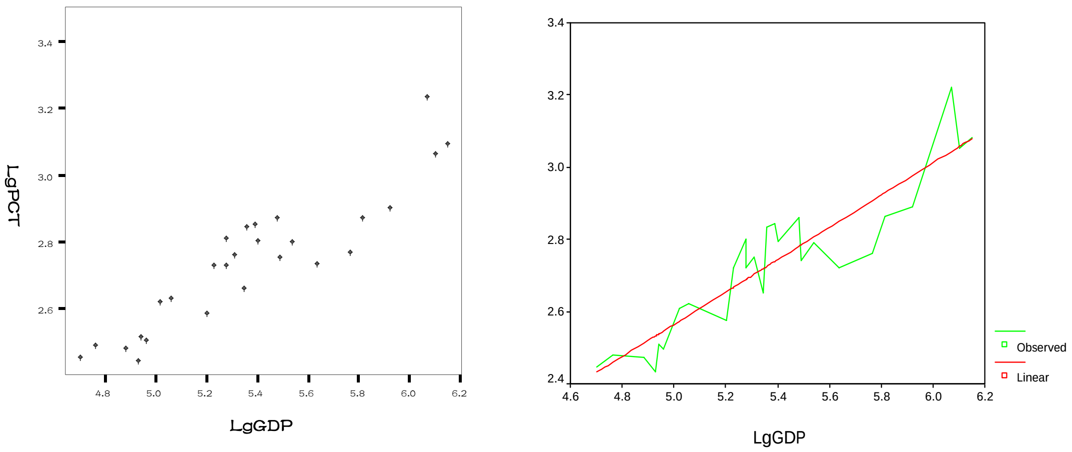

Figure 2: Correlation Analysis of Some Other Countries' PCT & GDP

The Relationship Between U.S. Patent Quantity & GDP
In order to examine whether the nonlinear power function correlation between patent output and GDP is a common phenomenon, we did same calculation with 1998-2003 US Patent data of some main developed countries (see to Table 2) and obtained results as below (see to Fig. 3):

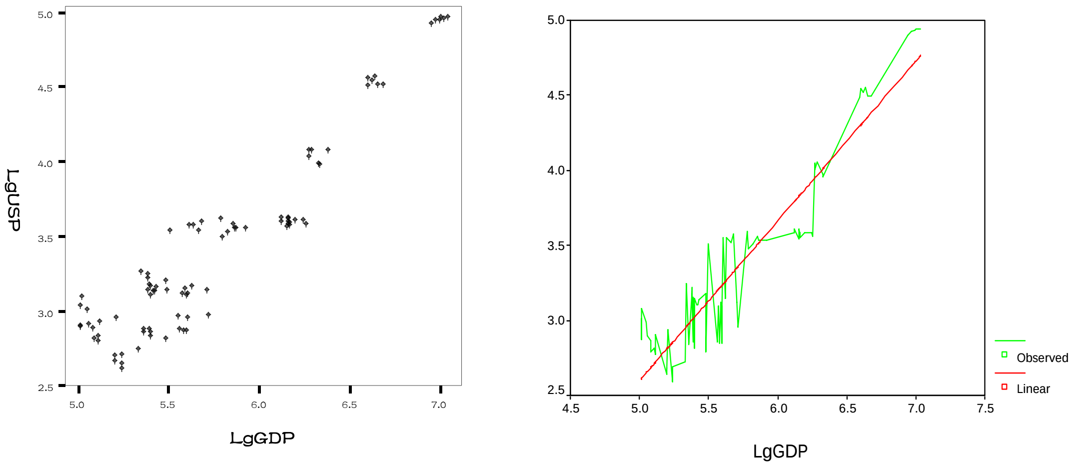

Figure 3: Correlation Analysis of Developed Countries' US Patent & GDP

Independent: Ln(x=GDP)

Dependent:	Mth	Rsq	d.f.	F	Sigf	a	b
Ln(Y=USP):	LIN	0.882	88	657.92	0.000	-27.210	10.639

With the outcome above, we know there is a strong correlation between main developed countries' US Patent output and relevant GDP as well, which is same as the case of world patent (PCT). It is a power function correlation, with the power index, b, almost equal to 1. This agrees with the qualitative conclusion of Prof. Price and Prof. Narin.

Power function is a nonlinear function. According to fractal theory (Mandelbrot, 1982, Feder, 1988), it is a fractal. Thus we could apply fractal theory when we describe and discuss about its property, characteristics and meaning.

Analysis & Discussion

Method & Data

Patent output is one of the most important measurements of technology innovation. However, it is no meaning to compare each country's own data of patent application and approval, because different countries have different patent laws and systems, thus have various standards and processes of applying for patent and obtaining approval. On the other hand, it is significant to evaluate data from same patent system, especially a system representing the international technical competitive environment, such as World Intellectual Property Organization (WIPO), or a system standing for the most advanced science and technology in the world, such as US patent system. These kinds of comparison can really reflect the distance of technological innovation ability and efficiency between countries. Because of this, we have chosen data from PCT and US patent with the aim of stating the relationship between economic situation and technology development on the international competitive background.

Besides, in the term of patent quality and in the microcosmic perspective, it is possibly correct that we should not treat all patents equally since different patents have different value to the society. However, from the view of macroscopic statistics, there is a normal distribution in one patent system, which means the bigger the patent quantity, the more the patents with high quality, and vice versa. Therefore, patent quantity, more or less, can objectively represent one country's technology innovation ability and efficiency. At the same time, we can use macroscopic statistics to eliminate some errors in order to make the result more accurate. This is the basic presupposition of our research here.

The Meaning of Power Index

In the nature, the most typical fractal with power function is the fractal growth in physics called mass fractal (Lin, Li, 1992), such as crystal growth in liquid, electrochemistry deposition, material fractal, and so on. There is a fractal function for fractal cluster's mass (M) and cluster's size (r): M(r) % r^{D_f}. D_f is called mass fractal dimension, which is an important index to reflect the balance of the fractal cluster's mass distribution. The bigger the D_f's value, the more the mass centralizes, and the less the D_f's value, the more the mass disperses. All mass fractals of physical objects have a geometry structure with self-similarity. On the other side, the quantitative power function of developed countries' patent quantity and GDP is a fractal without geometry structure, although the power index also is a fractal dimension (stated as D). For fractals without geometry structure, generally D is significant for describing fractal's characteristics, which implies D's amount is the degree of the fractal's centralization or dispersion (Li, Huang, Fang, 1999). Specifically, in the relationship between patent output and GDP, power index or fractal dimension, b = D = Ln(PCT)/Ln(GDP), is the technological innovation efficiency or density of patent to GDP. The bigger the D is, the higher the degree of fractal's centralization is, and the more the patents are produced on same GDP. For example, in the calculation above, the power index of developed countries (for PCT, D is about 0.74) is higher than that of some other countries (D is about 0.45). In the theory of Prof. Price and Prof. Narin, there is a direct proportion for scientific and technological output (patent, dissertation, etc.) and economic size, which is actually a case with D equal to 1. Furthermore, data of patent and GDP satisfy the rule of power fractal, which we consider as that the technological innovation ability or efficiency is relatively stable when the outside does not change very much.

On the other hand, power index or fractal dimension D also implied the characteristics of the fractal distribution, namely, the centralization and decentralization of the distribution or unbalanced degree. The bigger the D value, the more the unbalanced of the distribution, and the more the different of the technological innovation efficiency or level between countries, the more the unbalanced of the development of countries.

The point which we would like to clarify is that different data groups or objects have different fractal dimensions, although they all belong to same fractal, such as power fractal. For instance, compared with PCT, US patent data have a bigger power index or fractal dimension, and thus a higher centralization. The reason is that some American countries tend to apply for US patent, while European countries prefer to apply for PCT, and therefore the quantity of US patent might be smaller (of course some patents are PCT and US patent at the same time). Thus the slope in Figure 3 is bigger. In conclusion, power indexes of same patent data group can be compared, and power indexes of different patent data groups can not be simply compared directly.

The Explanation and Characteristics of Fractal

According to the calculation above, there is a strong power function correlation between developed countries' patent output and GDP. For the following countries with lower patent output, their data before 2001 can not satisfy a power function, but they do later since their patent quantity has increased (such as since 2001). We can understand this point well from the view of fractal.

Based on fractal theory, its basic characteristics are self-similarity (or self-affine) and scale invariance, which means fractals in the nature and society only exist in certain range, level, area or yardstick hierarchically, and follow the rule of 'the part is similar to the whole' in the range with scale unchanged. Developed countries have high GDP and R&D investment (the R&D investment of countries with highest patent output is over 1.8% of their GDP), and thus high technological innovation foundation and efficiency. Although there is still difference among them, the difference is on the quantity aspect instead of the essence aspect. Hence, these countries are in the same group/cluster or on the same level, and satisfy same regulation, which is that one of them has similar technological innovation efficiency as the others. In addition, from the view of statistics, the patent output of these developed countries is over 90% of the patent output in the world. Such a large quantity is very significant and necessary to the statistics calculation.

On the other hand, the R&D investment of the following developing countries is only about 1% of their GDP or even less, so their technological innovation efficiency is lower. When their technological innovation has reached certain level and their patent quantity has increased to certain amount which is needed for a statistics sample, a power fractal correlation may show out. With their own fractal dimension, these countries belong to another level and their situations are certainly similar. If one or some countries of them have increased its/their R&D investment to 1.8% of its/their GDP or even more, and its/their technological innovation ability has improved, this country or these countries are able to enter the level of developed countries. For example, since 1990, Korea had rapidly raised its R&D investment and gradually closed to the level of developed countries. After some years, Korean PCT quantity ranked at 11 in the world in 2000. As a result, Korea had entered the group of developed countries. Therefore, with this index, we are able to examine one country's economy, technology and society development.

Conclusion

There is a quantitative power function correlation between patent output and GDP. This relationship is a power fractal, which is a valuable discovery. With it, we are able to predict one country's patent quantity or technological innovation efficiency through analyzing its GDP, and vice versa. Meanwhile, we can judge whether one country has entered one group or level by checking whether its PCT and GDP data can satisfy the power function correlation of that group or level. The amount of power index represents the level of technological innovation efficiency. And the technological innovation ability is hierarchical. Countries in the same group follow same regulations and similar to each other. Also, we have studied the distribution of Province/area 's China Patent Number and Province/area GDP, and found that there is power law function, a Fractal, which would be published on another paper. However, we do not know clearly whether the power fractal correlation between patent output and GDP is a common phenomenon, and what is its essence and internal system. Hence, we need more data to process more experiments and discussion in the future. Finally, according to our result above, we would conjecture that there would be the power function relation between the publication(papers) and GDP for countries.

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